Factors Influencing Power

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Introduction

- In the preceding lecture, we examined hypothesis testing as a general strategy.
- Then, we examined how to calculate critical values, power, and required sample size in the specific case of a hypothesis test on a single mean, when σ is somehow known.

Introduction

- At the end of the lecture, we introduced the notion of *standardized effect size* E_s, the amount by which the null hypothesis is wrong in standard deviation units.

$$E_s = \frac{\mu - \mu_0}{\sigma} \tag{1}$$

- We found that power can be calculated from the simplified formula

$$\text{Power} = \Phi(\sqrt{n}E_s - Z_{crit}) \tag{2}$$

where Φ is the normal curve cumulative probability (`pnorm` in R) and Z_{crit} is the critical value for the Z-statistic.

Introduction

- We then worked with the formula showing that the sample size n required to achieve a desired level of power can be calculated directly as

$$n_{required} = \text{ceiling}\left(\left(\frac{Z_{crit} + Z_{power}}{E_s}\right)^2\right) \tag{3}$$

where Z_{power} is the normal curve quantile corresponding to the desired level of power.

Introduction

- In this module, we want to re-examine these formulas to see what they can tell us about
  1. The factors influencing power of a hypothesis test
  2. The factors influencing the n required to achieve a given level of power
- Let's begin by examining the factors influencing power.

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- As we saw in Equation 2, power is evaluated by computing $\Phi(K)$, where $K = \sqrt{n}E_s - Z_{crit}$.
- The normal curve cumulative probability function Φ is a strictly increasing S-shaped function, as shown in the plot on the next slide.
- So anything that increases K must also increase power, all other things being equal. Likewise, anything that decreases K must also decrease power.

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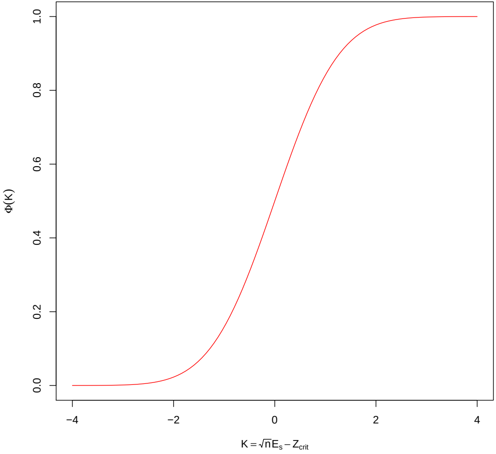

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- Since n and E_s and Z_{crit} are all assumed to be positive when this formula is used, it is clear that K will increase as a function of n and E_s, while it will decrease as a function of Z_{crit}. So, all other things being equal,
  1. **As n increases, power increases.** This is true because the left side of the formula for K increases as a function of the square root of n.
  2. **As E_s increases, power increases.** Two factors can cause E_s to increase. (1) The null hypothesis can become more false because of an increased experimental effect (due to, for example, a more powerful manipulation on the part of the experimenter), or (2) The natural variation σ in the measurement of the dependent variable decreases because of refinements in the measurement method or a decrease in variability in the population.
  3. **As α increases, power increases.** This is because increasing α allows the critical value Z_{crit} to decrease, thereby reducing K in the plot. For example, "relaxing" α from 0.01 to the "less stringent" 0.05 will increase power.
  4. **Shifting from a 2-tailed test to a correctly specified 1-tailed test with the same α will increase power.** This is because shifting from a 2-tailed test to a 1-tailed test will move half the rejection area to the other rejection region. This will decrease Z_{crit} and increase power. On the other hand, of course, shifting from a 1-tailed test to a 2-tailed test will cost you power.

Factors Influencing Required n

- Since n and power are directly related, we can conclude that other factors we discussed that increase power will, all other things being equal, decrease the required sample size n.
- Another way of examining the issue is to re-examine the formula for required n, i.e.,

$$n_{required} = \text{ceiling}\left(\left(\frac{Z_{crit} + Z_{power}}{E_s}\right)^2\right) \tag{4}$$

- Anything increasing either term in the numerator will increase required n, and anything decreasing E_s will increase the required n.
- Decreasing ("tightening") α or switching from a 1-tailed to a two-tailed test will increase Z_{crit}. Increasing required power will increase Z_{power}.
- Any decrease in the experimental effect or increase in σ will decrease E_s.
- Taking these facts into consideration, we emerge with the rules summarized on the next slide.

Factors Influencing Required n

All other things being equal,

- **Increasing ("relaxing")** α will **decrease** the required n.
- **Switching** from a 2-tailed test to a correctly specified 1-tailed test will **decrease** the required n.
- **Increasing** the experimental effect (with a stronger manipulation) will increase E_s and **decrease** the required n.
- **Decreasing** the natural variation in measurements will increase E_s and **decrease** the required n.
- **Decreasing** the required power will **decrease** the required n.

These Rules Generalize!

- In the previous sections, we developed rules of thumb for how various factors affect power and/or required sample size (n).
- We developed the rules by a careful consideration of the formulas for calculating power and required n in the context of the 1-sample Z test for a single mean.
- However, we will find that these results generalized across a wide variety of statistical tests.
- In the next module, we extend the 1-sample Z test to the situation in which the population standard deviation σ is not known.